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Exact solution of generalized t–J models in one dimension

R Z Barievt

Laboratoire de Physique du Solide, URA CNRS 155, Université de Nancy I, BP 239, F-54506 Vandœuvre lès Nancy cedex, France

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Abstract. A relation between the anisotropic t–J model and a generalized six-vertex model has been established. On the basis of this relation, we obtain the exact Bethe ansatz solution for the ground state energy of anisotropic t–J model for arbitrary spin and special values of coupling parameters. We consider also the anisotropic generalization of t–J model with localons.

1. Introduction

The discovery of high-temperature superconductors has greatly stimulated the interest in strongly correlated systems. Recently there has been a renewed interest in the one-dimensional t–J model as an integrable low-dimensional version of a strongly correlated electronic system. Zhang and Rice [1] have shown that a wide class of extended Hubbard models can be suitably reduced to the t–J model. Anderson [2] claimed that due to the strong quantum fluctuations in low dimensions, one and two dimensions may have common aspects, so that it is very important to understand exact results available in one dimension.

This model describes electrons with nearest-neighbour hopping, with the constraint that two electrons are not allowed to simultaneously occupy the same site. This restriction corresponds to an implicitly infinite on-site Coulomb repulsion. Two types of interactions between electrons on nearest-neighbour sites are considered: a charge interaction of strength V and a spin-exchange interaction J. The Hamiltonian of the extended version of the t–J model has the form [3, 4]

$$\mathcal{H} = -\sum_{j,s} P\left(c_{js}^{+}c_{j+1s} + c_{j+1s}^{+}c_{js}\right)P + J\sum_{j,s,s',s\neq s'} c_{js}^{+}c_{js'}\cdot c_{j+1s'}^{+}c_{j+1s} + \sum_{j,s,s'} V_{ss'}c_{js}^{+}c_{js}c_{j+1s'}^{+}c_{j+1s'} \tag{1}$$

where c_{js} annihilates an electron with spin component s. We assume for convenience that $s = 1, \ldots, 2S+1$, P is the projector on the subspace of non-doubly occupied states. We have introduced an anisotropy in the charge interactions through a matrix $V_{s,s'}$.

In the isotropic case $V_{s,s'} = V$ the Hamiltonian (1) corresponds to the traditional t–J model which was exactly solved by the Bethe ansatz method [3, 5–7] for the values J and V corresponding to the supersymmetric situation ($V = \pm J = \pm 1$) and $S = 1/2$. The critical exponents of this model were calculated in [8]. The generalization of above-mentioned

† Permanent address: The Kazan Physico-Technical Institute of the Russian Academy of Sciences, Kazan 420029, Russia.

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results for arbitrary spin S was carried out in [4, 9, 10]. Other generalizations of the t–J model were studied by Klümper *et al* [11] and Essler *et al* [12, 13].

In this paper we consider an anisotropic generalization of the t–J model (1) and show that it is soluble for arbitrary spin and special values of the couplings J and $V_{s,s'}$. We consider also an anisotropic t–J model with localons for $S = 1/2$. The isotropic version of the last model was proposed and solved exactly in [12]. The exact solution for the ground state energy is presented for the one-dimensional anisotropic t–J model with spin $S = 1/2$.

2. Perk–Schultz model

Our starting point is the multi-component generalized six-vertex model, also called Perk–Schultz model [14, 15], which was diagonalized by Schultz [16] in the most general form. Some interresting applications of this model in quantum field theory were considered by de Vega and Lopes [17].

Schultz [16] considered a two-dimensional square lattice with the variable on the lattice bonds taking on values $0, 1, \ldots, q-1 = 2S+1$. Each type of configuration of four bonds meeting at a vertex has associated with it an energy and a corresponding Boltzmann weight $R^{\mu\nu}_{\alpha\beta}$, see figure 1. In general, there are q^4 types of vertices. The model under consideration is defined by a one-parameter family of vertex weights $R^{\mu\nu}_{\alpha\beta}(u)$, where the non-vanishing elements of the R-matrix are

$$\begin{aligned} R^{\alpha\alpha}_{\alpha\alpha}(u) &= \sinh(\eta + \epsilon_\alpha u)/\sinh\eta \\ R^{\mu\mu}_{\alpha\alpha}(u) &= \varepsilon_{\alpha\mu}\sinh u/\sinh\eta \\ R^{\mu\alpha}_{\alpha\mu}(u) &= \exp[u\ \mathrm{sign}(\mu-\alpha)] \end{aligned} \tag{2}$$

where $\alpha, \mu = 0, 1, 2, \cdots, q-1$ and $\alpha \neq \mu$, with $\varepsilon_{\alpha\mu} = \varepsilon_{\mu\alpha} = \pm 1$ and $\epsilon_\alpha = \pm 1$. The weights (2) are a solution of the Yang–Baxter equations [18], which ensures integrability.

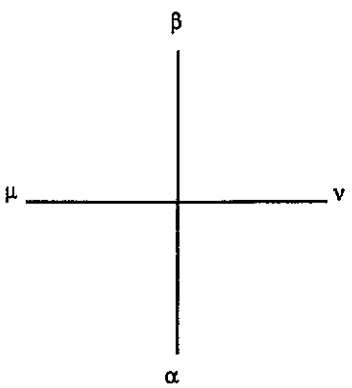

Figure 1. Vertex configuration of the Perk–Schultz model.

A quantum Hamiltonian for an integrable $SU(2S+2)$ spin chain can be obtained through the well known relation [18]

$$\mathcal{H} = \sinh\eta\ \frac{\mathrm{d}}{\mathrm{d}u}\{\ln[T(u)]\}\Big|_{u=0} = \sinh\eta\ T(0)^{-1}\left[\frac{\mathrm{d}T(u)}{\mathrm{d}u}\right]\Big|_{u=0} \tag{3}$$

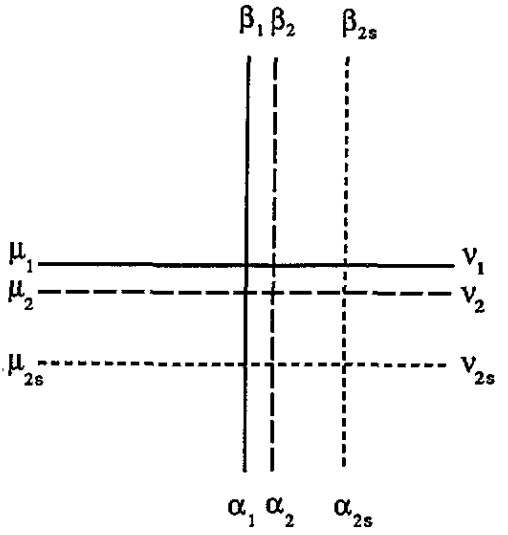

Figure 2. Vertex configuration of compositive lattice.

where $T(u)$ is the row-to-row transfer matrix

$$T(u)^{\{\beta\}}_{\{\alpha\}} = \sum_{\{\mu\}} \prod_{j=1}^{N} R^{\mu_j \mu_{j+1}}_{\alpha_j \beta_j}(u) \qquad \mu_{N+1} \equiv \mu_1 \tag{4}$$

and N is the number of sites per row.

At $q = 2$ ($s = 0$) equations (2) determine the ordinary six-vertex model and equation (3) gives the Hamiltonian XXZ chain with spin $\tilde{S} = 1/2$ ($\tilde{S} = S + 1/2$) [18]. For arbitrary q we obtain a generalization of these results to the case of spin $\tilde{S} > 1/2$ [14]. In the isotropic limit ($\eta \to 0$, $u \to 0$ but u/η is fixed) the trigonometric (hyperbolic) solution (2) reduces to the rational one and equation (3) gives the Hamiltonian of the Lai–Sutherland model [4, 5]. The proof of the equivalence of this model to the isotropic t–J model at the supersymmetrical point is based on the fact that the Hamiltonians of both models can be presented in terms of a graded permutation operator [19, 20]. This approach is not available for the trigonometrical solution (2) and the purpose of this paper is to address two points: (1) What is the generalization of the t–J model associated with Perk–Schultz solution (2) or with spin Hamiltonians which were obtained from this solution? (2) What is the exact solution of this generalized t–J model?

3. Anisotropic t–J model with arbitrary spin

In contrast to [14], we consider the lattice with weight (2) as a composite one, which consists of $q - 1$ sublattices (see figure 2). The edges of each sublattice have two possible states, they may be either empty ($\alpha = -1$) or occupied ($\alpha = +1$). Then each composite edge of the original lattice has 2^q possible states but we exclude states involving double occupancy. In this case we have only q possible states for the edges of the original lattice:

$$\alpha = (\alpha_1, \ldots, \alpha_{q-1}) = (-1, \ldots, -1); (+1, -1, \ldots, -1); \ldots, (-1, \ldots, -1, +1).$$

In this way we may consider the original model in the spirit of [21] as a multi-sublattice system with interactions between sublattices of the vertex–vertex type. Each sublattice corresponds to the six-vertex model. The interactions between the sublattices lead to a change in the Boltzmann weights of the composite system which are no longer products of the sublattice Boltzmann weights but are given by (2). In this equation we will now consider the indices α, β, μ, ν as $(q-1)$-component quantities, e.g.

$$\alpha = (\alpha_1, \ldots, \alpha_{q-1}) \qquad \beta = (\beta_1, \ldots, \beta_{q-1})$$
$$\mu = (\mu_1, \ldots, \mu_{q-1}) \qquad \nu = (\nu_1, \ldots, \nu_{q-1}).$$

Then the vertex weights have the form

$$\begin{aligned} R^{\{\mu_l\}\{\nu_l\}}_{\{\alpha_l\}\{\beta_l\}}(u) = \frac{1}{2^{2N}} \prod_{s=1}^{q-1}(1-\mu_s)(1-\alpha_s) \Bigg\{ \Bigg[R^{00}_{00}(u) + \sum_{s=1}^{q-1} t(\mu_s)t(\alpha_s) R^{ss}_{ss}(u) \\ + \sum_{s=1}^{q-1} \left(t(\mu_s)\mathrm{e}^{u} + t(\alpha_s)\mathrm{e}^{-u} \right) + \sum_{\substack{s,s'=1 \\ s\neq s'}}^{q-1} t(\mu_s)t(\alpha_{s'})\mathrm{e}^{\mathrm{sign}(s-s')u} \Bigg] \\ \times \prod_{s=1}^{q-1} \delta(\mu_s, \beta_s)\delta(\alpha_s, \nu_s) \\ + 2\left(\frac{\sinh u}{\sinh \eta}\right) \Bigg[\sum_{s=1}^{q-1} \varepsilon_{s0} T(\mu_s, \alpha_s)\delta(\mu_s, -\beta_s)\delta(\alpha_s, -\nu_s) \prod_{\substack{l=1 \\ l\neq s}}^{q-1} \delta(\mu_l, \beta_l)\delta(\alpha_l, \nu_l) \\ + \sum_{s<s'} \varepsilon_{ss'}(1-\mu_s\mu_{s'}) T(\mu_s, \alpha_s) T(\mu_{s'}, \alpha_{s'}) \prod_{\substack{l=1 \\ l\neq s,s'}}^{q-1} \delta(\mu_l, \beta_l)\delta(\alpha_l, \nu_l) \\ \times \prod_{l'=s,s'} \delta(\mu_{l'}, -\beta_{l'})\delta(\alpha_{l'}, -\nu_{l'}) \Bigg] \Bigg\} \end{aligned} \tag{5}$$

where

$$t(\mu) = \frac{1+\mu}{1-\mu} \qquad T(\mu, \alpha) = \frac{1-\mu\alpha}{(1-\mu)(1-\alpha)}$$

Substituting the form (5) into (3) and using the standard definition of Pauli spin operators, one obtains the Hamiltonian of the $(q-1)$ sublattice spin model

$$\begin{aligned} \mathcal{H} &= \sum_{j=1}^{N} \mathcal{H}_{j,j+1} \\ \mathcal{H}_{j,j+1} &= \sum_{s=1}^{q-1} \varepsilon_{s0}(\sigma^+_{j,s}\sigma^-_{j+1,s} + \sigma^+_{j+1,s}\sigma^-_{j,s}) \\ &\quad + \sum_{\substack{s,s'=1 \\ s\neq s'}}^{q-1} \varepsilon_{s,s'}\sigma^+_{j,s}\sigma^-_{j+1,s}\sigma^+_{j+1,s'}\sigma^-_{j,s'} + \cosh\eta \sum_{s=1}^{q-1} (\epsilon_0 + \epsilon_s) n_{js} n_{j+1,s} \\ &\quad + \epsilon_0 \sum_{\substack{s,s'=1 \\ s\neq s'}}^{q-1} \exp\left\{\epsilon_0 \mathrm{sign}[(s-s')\eta]\right\} n_{j,s} n_{j+1,s'} + \epsilon_0 \left(\cosh\eta - \mathrm{e}^{-\epsilon_0\eta} n_j - \mathrm{e}^{\epsilon_0\eta} n_{j+1}\right) \end{aligned} \tag{6}$$

where n_j is the density operator

$$n_j = \sum_{s=1}^{q-1} n_{js} \qquad n_{js} = \sigma_{j,s}^{+}\sigma_{j,s}^{-}$$

N is the number of lattice sites. The constraint imposed on spins is that the configurations which contain more than one spin up on each lattice site are strictly prohibited. Due to this constraint we have omitted some terms in equation (6), which contain more than one operator σ_{js} with the same index j.

Using a Jordan–Wigner transformation [22], we obtain a Hamiltonian of the generalized t–J model

$$\begin{aligned}\mathcal{H} = &-\sum_{j,s} \varepsilon_{s0} P(c_{j,s}^{+}c_{j+1,s} + c_{j+1,s}^{+}c_{j,s})P \\ &- \sum_{\substack{j,s,s' \\ s\neq s'}} \varepsilon_{s,s'} c_{j,s}^{+}c_{j,s'}c_{j+1,s'}^{+}c_{j+1,s} + \cosh\eta \sum_{j,s}(\epsilon_0 + \epsilon_s) n_{js}n_{j+1,s} \\ &+ \epsilon_0 \sum_{\substack{j,s,s' \\ s\neq s'}} \exp\left\{\epsilon_0 \operatorname{sign}[(s-s')\eta]\right\} n_{j,s}n_{j+1,s'} \\ &+ \epsilon_0 \sum_{j} \left(\cosh\eta - \mathrm{e}^{-\epsilon_0\eta} n_j - \mathrm{e}^{\epsilon_0\eta} n_{j+1}\right) \end{aligned} \tag{7}$$

with

$$n_j = \sum_s n_{js} \qquad n_{js} = c_{j,s}^{+}c_{j,s} \qquad \sigma_{js}^{+} = \exp\left(-\mathrm{i}\pi \sum_{k=1}^{j-1} c_{ks}^{+}c_{ks} - \mathrm{i}\pi \sum_{l=1}^{s-1}\sum_{k=1}^{N} c_{kl}^{+}c_{kl}\right) c_{js}.$$

Equation (7) determines the integrable cases of the Hamiltonian (1).

The exact solution for the total energy of the Hamiltonian (7) can be obtained from the Schultz paper [13] and is given by

$$E = \epsilon_0 \left\{ N\cosh\eta - 2\sinh^2\eta \sum_{j=1}^{N_e} \left[\cosh\eta - \cos\lambda_j^{(0)}\right]^{-1} \right\} \tag{8}$$

$$\begin{aligned}\prod_{s=0}^{q-1} \left(\varepsilon_{sl}\varepsilon_{l+1,s}\right)^{N_s} \epsilon_l^{N_l}\epsilon_{l+1}^{N_{l+1}} \prod_{j'=1}^{M_{l-1}} \frac{\sin(\lambda_j^{(l)} - \lambda_{j'}^{(l-1)} + \mathrm{i}\epsilon_l\eta/2)}{\sin(\lambda_j^{(l)} - \lambda_{j'}^{(l-1)} - \mathrm{i}\epsilon_l\eta/2)} \\ = -\prod_{j'=1}^{M_l} \frac{\sin(\lambda_j^{(l)} - \lambda_{j'}^{(l)} + \mathrm{i}\epsilon_{l+1}\eta)}{\sin(\lambda_j^{(l)} - \lambda_{j'}^{(l)} - \mathrm{i}\epsilon_l\eta)} \prod_{j'=1}^{M_{l+1}} \frac{\sin(\lambda_j^{(l)} - \lambda_{j'}^{(l+1)} - \mathrm{i}\epsilon_{l+1}\eta/2)}{\sin(\lambda_j^{(l)} - \lambda_{j'}^{(l+1)} + \mathrm{i}\epsilon_{l+1}\eta/2)} \\ j = 1, \ldots, M_l \qquad l = 0, 1, \ldots, q-2 \end{aligned} \tag{9}$$

where

$$\lambda_j^{(-1)} = 0 \qquad \varepsilon_{l,l} = 1 \qquad M_{-1} = N \qquad M_{q-1} = 0 \qquad N_j = M_{j-1} - M_j$$

N_{q-m} is the number of electrons with spin component m, N_0 is the number of empty sites.

To the author's knowledge, the Hamiltonian (7) includes as particular cases all integrable generalizations of t–J model considered previously (see, however, [11]). In particular, in the isotropic limit for the $\varepsilon_{s0} = 1$, $\varepsilon_{s,s'} = \epsilon_0 = -\epsilon_s$ the Hamiltonian (7) yields the integrable $SU(2S+2)$ generalization of the t–J model solved by Lee and Schlottman [4] (see also [9, 10]).

4. Anisotropic t–J model with localons

Next we consider the anisotropic generalization of the t–J model with localons for the spin $S = 1/2$. The isotropic version of this model was proposed and solved exactly by Essler *et al* [12]. The exact solution [12] is made possible by the fact that hopping processes interfere in such a way that the number of doubly occupied sites called localons is conserved. Thus in this model there are three types of excitations and we must consider the solution of Yang–Baxter equations (2) at $q = 4$ for the two-sublattice system i.e. add states involving double occupancy. We suppose also that the number of localons is conserved in each lattice site as ordinary arrows in the six-vertex model [18]. Then the above procedure gives the following Hamiltonian:

$$\mathcal{H} = \sum_{j=1}^{N} \mathcal{H}_{j,j+1}$$

$$\begin{aligned}\mathcal{H}_{j,j+1} = &- \sum_{\substack{s=1,2 \\ s' \neq s}} \varepsilon_{s0}\left[1 - n_{js} - n_{j+1,s} + (1 + \varepsilon_{s0}\varepsilon_{s3}) n_{js} n_{j+1,s}\right](c^{+}_{js'} c_{j+1,s'} + \text{H.C.}) \\ &+ \left(\varepsilon_{30} c^{+}_{j1} c_{j+1,1} c^{+}_{j2} c_{j+1,2} - \varepsilon_{12} c^{+}_{j1} c_{j2} c^{+}_{j+1,2} c_{j+1,1} + \text{H.C.}\right) \\ &+ \sum_{\substack{s=1,2 \\ s' \neq s}} \left[\cosh\eta(\epsilon_0 + \epsilon_s) n_{js} n_{j+1,s}(1 - n_{js'} - n_{j+1,s'}) + \right. \\ &\left. + \epsilon_0 \mathrm{e}^{(s-s')\eta}(n_{js} n_{j+1,s'} + n_{js} n_{js'})\right] \\ &+ 2\sinh\eta(n_{j1} n_{j2} n_{j+1,2} - n_{j2} n_{j+1,1} n_{j+1,2}) + \left(\sum_{l=0}^{3} \epsilon_l\right) n_{j1} n_{j2} n_{j+1,1} n_{j+1,2} \\ &+ \epsilon_0\left[\cosh\eta - \mathrm{e}^{-\epsilon_0\eta} n_j - \mathrm{e}^{\epsilon_0\eta} n_{j+1}\right]. \end{aligned} \tag{10}$$

The exact solution for the total energy of the Hamiltonian (10) is given by equations (8)–(9) at $q = 4$ where N_3 is the number of localons.

The Hamiltonian (10) in the isotropic limit ($\eta \to 0$) for the particular choice

$$\varepsilon_{s0} = -\varepsilon_{s3} = -\varepsilon_{30} = -\varepsilon_{12} = 1 \qquad \epsilon_0 = \epsilon_3 = -\epsilon_s = -1 \qquad (s = 1, 2)$$

yields the t–J model with localons which was considered by Essler *et al* [12]. For other choices of ϵ_i we obtain other integrable cases which correspond to different gradings of the model. It is noteworthy that equations (8)–(9) ensure that we are able to obtain the complete exact solution of the problem under consideration. However, in some cases this problem is not simple and equations (9) have to be solved numerically.

5. Ground-state energy

In conclusion we present the exact solution for the ground-state energy of the anisotropic t–J model (7) for the $S = 1/2$ spin in the physically more relevant case $\varepsilon_{s0} = 1$, $\varepsilon_{s,s'} = \epsilon_0 = -\epsilon_s = -1$ $(s, s' = 1, 2)$. To obtain a closer relation with previous work [3, 5–7], we omit the last term in equation (7). It leads to only a trivial shift of $-\epsilon_0 N_0 \cosh\eta$ in energy. In general, the ground state consists of a certain number of singlet bound pairs and a certain number of free electrons with the same spin. But in the absence of an external

magnetic field, there are equally many electrons with up and down spin, and the ground state contains only bound pairs characterized by a pair of complex electron rapidities

$$\lambda_\alpha^{\pm} = \frac{1}{2}(v_\alpha \pm i\eta) \qquad v_\alpha = 2\Lambda_\alpha. \tag{11}$$

Using (11), equations (8)–(9) are reduced to the following set of equations after some simple algebra:

$$E = \sum_{\beta=1}^{M} e_0(v_\beta) \qquad P = \sum_{\beta=1}^{M} [\pi - \Theta(v_\beta; \eta)]$$

$$e_0(v) = -4\cosh\eta + 2\sinh\eta\,\Theta'(v;\eta) \tag{12}$$

$$\Theta(v;\eta) = 2\arctan(\coth\eta\tan v/2) \qquad -\pi < \Theta(v;\eta) \leqslant \pi$$

$$N\Theta(v_\beta;\eta) - \sum_{\alpha=1}^{M} \Theta(v_\beta - v_\alpha;\eta) = 2\pi J_\beta \tag{13}$$

where $M = N_1 = N_2$ is the number of bound pairs, P is the momentum of the state under consideration, J_α are integer (half-integer) numbers for even (odd) $M+1$. They are restricted to the interval $|J_\alpha| \leqslant J_{\max} = (N - M - 1)/2$ [7]. At half-filling, the number of available positions is $N/2$, so that there is no freedom in distributing the numbers J_α. Away from half-filling, the number of available states exceeds the number of actual pairs, so that freedom is left in the choice of the J_α's. The ground state energy is obtained by choosing $|J_\alpha|$ as close as possible to $J_{\max}$.

In the thermodynamic limit $N \to \infty$, $M \to \infty$ for a fixed ratio M/N, the value v_β fill the intervals $[-\pi, -v_0]$ and $[v_0, \pi]$ uniformly with density $\sigma(v)$. From (13) we obtain an integral equation for the distribution function $\sigma(v)$,

$$\sigma(v) = \frac{1}{2\pi}\Theta'(v;\eta) - \left[\int_{-\pi}^{-v_0} + \int_{v_0}^{\pi}\right] \frac{dv'}{2\pi}\Theta'(v - v';\eta)\,\sigma(v')\,dv' \tag{14}$$

with the subsidiary condition

$$\left[\int_{-\pi}^{-v_0} + \int_{v_0}^{\pi}\right] \sigma(v)\,dv = M/N \tag{15}$$

where $2\rho = 2M/N$ is the density of the electron liquid.Thus the ground state energy can be deduced from the solution of the integral equation (14) as

$$E = 2\cosh\eta\,(1 - 2\rho) - 4\pi\sinh\eta\sigma(0) \tag{16}$$

and is a function of the filling ρ. At half-filling $M = N/2$, the solution of (14)–(16) can be obtained in a closed form:

$$E = -\sinh\eta\left[1 + 4\sum_{m=1}^{\infty} \frac{1}{1 + \exp(2m\eta)}\right]. \tag{17}$$

In the isotropic limit ($\eta \to 0$), the ground-state energy is $E = -2\ln 2$ in agreement with references [3, 7, 23].

Away from half-filling, the integral equation has to be solved numerically. For all filling values, the ground state is a liquid of singlet bound pairs. In contrast to the isotropic t–J model [3, 7], these pairs have a non-zero binding energy, so that a threshold magnetic field is required to overcome it. Therefore, these pairs are Cooper-like bound pairs.

A more detailed study of the superconductivity properties of the model is certainly needed and will be the subject of future investigations.

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